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A mechanistic model for heat transfer from a wall to a fluid

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Abstract—The mechanism of heat transfer from a wall to a fluid in a turbulent boundary layer is discussed. A simplified model of heat transfer in the near-wall region is proposed, taking into account bursting phenomenon. The effect of bursts on the rate of heat transfer from a solid wall to the fluid is estimated.

INTRODUCTION

The standard analysis of heat transfer in turbulent boundary layer (e.g. see Gröber *et al.* [1], Kays [2], Bejan [3], etc.), is based on simplified models of turbulence which do not account for any mechanisms of near-wall flow. This does not allow one to reveal the true mechanism of heat removal in turbulent boundary layer, and makes it difficult to propose a rational theory of this process.

The investigations done during the last decades (beginning with Kline *et al.* [4]) showed that near-wall flow possesses a rather complicated structure which results from strong interaction between large-scale vortices emerging in turbulent boundary layer, and low-speed streaks existing in its sublayer. This process is accompanied by burst formation leading to enhancement of heat removal from the wall. There are two key issues which arise: (i) what is the real mechanism of heat removal from a solid wall to the fluid in turbulent boundary layer and (ii) what is the role played by bursts in this mechanism and what is its effect on the rate of heat removal? An associated issue to the latter one is: can heat transfer from a solid wall to fluid flowing over it be modulated through, say, modulation of the bursting frequency? Experimental evidence of the events taking place in the near wall region allow one to develop a more realistic (from the hydrodynamic point of view) scenario of the heat removal in a turbulent boundary layer.

We propose a new approach to the analysis of heat transfer in a turbulent boundary layer, and aim to answer the above questions. In this paper we focus on the mechanism of heat removal from the wall in a turbulent boundary layer. Our interpretation of this phenomenon is based on the assumption of a dominant role of the bursts, leading to formation of zones with very small thermal resistance in the sublayer of the turbulent boundary layer. We also discuss briefly some details of the description of heat removal in turbulent boundary layer accounting for bursting process as described by Kaftori *et al.* [5].

FUNNEL VORTICES

It is well known that the viscous sublayer plays an important role in the transport processes occurring in the near-wall flow. Destruction of this layer at the moment of burst formation leads to a drastic change in the conditions of heat removal from the wall. Burst formation leads to a decrease in the thickness of the sublayer which, in its turn, decreases thermal resistance in the domain where the burst is born. As a consequence, in the sublayer of turbulent boundary layer some zones have very little thermal resistance, and the heat is carried away from the wall by funnel vortices or by vigorous bursting. We assume that the amount of heat removed from the wall in turbulent boundary layer is determined primarily by these coherent structures.

The structure of turbulent boundary layer in the domain of burst formation is shown schematically in Fig. 1. The fluid moving to the wall in the peripheral zone of burst has a temperature close to the temperature of the free stream T_∞. The fluid moving in the central zone of burst has the temperature T_m close to the temperature of the wall T_w. The nonuniform distributions of temperature T and transversal component of velocity v in the cross-section of the burst are due to its interaction with the wall and surrounding fluid. These distributions may be presented in the following form: $v/v_m = f(\eta)$, $\Delta T/\Delta T_m = \varphi(\eta)$ where f and φ are some functions of the variable η; $\eta = \xi/\xi_o$, ξ_o and ξ are the radius of burst and current radius, respectively, subscript m corresponds to the burst axis. It should be stressed that the shape of the burst assumed here is quite arbitrary. The results which are obtained below are quite independent of this shape.

HEAT TRANSFER COEFFICIENT

Consider a non-gradient flow of viscous incompressible fluid along the x-axis (Fig. 1). The velocity variation at a fixed point of the near-wall region of turbulent boundary layer is shown in Fig. 2, as a

NOMENCLATURE

A	the constant in equation (8)	t	time between bursts
B	the constant in equation (14)	t_1	duration of the quasi-laminar flow
b	the constant in equation (11)	t_2	burst duration
c_p	specific heat of the fluid at constant pressure	T_m	fluid temperature at the burst axis
ΔT	difference of the local fluid temperature and fluid temperature at the outer boundary of the boundary layer	T_w	temperature of the wall
ΔT_m	difference of the local fluid temperature at the burst axis and that of the outer boundary of the boundary layer	T_∞	fluid temperature at the outer boundary of the boundary layer
I	the integral in equation (7)	u, v	longitudinal and transversal components of fluid velocity in the boundary layer
L	characteristic length scale	u_∞	fluid velocity at the outer boundary of the boundary layer
n	the power in equation (8)	v_m	transversal velocity of fluid at the burst axis
Nu	the Nusselt number	x, y	longitudinal and transversal Cartesian coordinates
Nu_l	the Nusselt number in quasi-laminar flow	x_b	coordinate of burst location.
Nu_x	local value of the Nusselt number		
Nu_{xl}	local value of the Nusselt number in quasi-laminar flow	Greek symbols	
Pe_x	local value of the Peclet number	α	thermal transfer coefficient
Q	total amount of heat removed from the wall during the time between bursts	β	the constant in equation (13)
Q_1	amount of heat removed from the wall during period of quasi-laminar flow	γ	the ratio of the time of burst duration to the time between bursts
Q_2	amount of heat removed from the wall during burst	ε	the constant in equation (14)
Re	the Reynolds number	η	nondimensional coordinate
Re_x	local value of the Reynolds number	λ	thermal conductivity of the fluid
s	cross sectional area of the burst	ν	kinematic viscosity of fluid
		ξ	current radius of burst
		ρ	density of fluid
		τ	time.

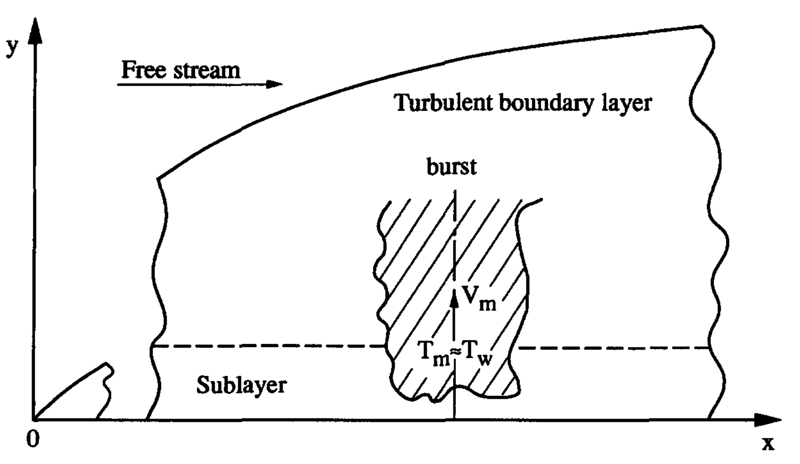

Fig. 1. Schematic description of a turbulent boundary layer.

function of time. This graph illustrates one of the most important features of the near-wall flow: existence of two distinct forms of fluid motion corresponding to the quasi-laminar and bursting flow occurring at distinct periods of time t_1 and t_2, respectively. At the first period the value of v/u (v and u are the longitudinal and transversal components of velocity) is smaller than one; at the second period these components have the same order of magnitude ($v \sim u$) and the ratio v/u is of order one.

To estimate the heat transfer from the wall to the fluid, we use a thermal balance for a small area s equal

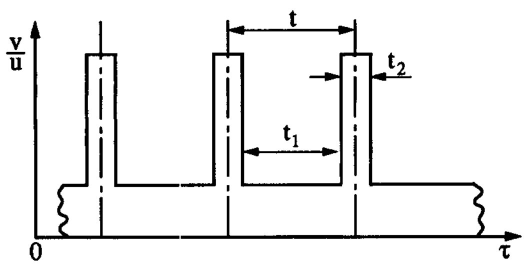

Fig. 2. Velocity variation at a fixed point in the sublayer.

to the area of the cross-section of a burst. In the quasi-stationary approximation the balance equation is:

$$Q = Q_1 + Q_2 \tag{1}$$

where Q is the total amount of heat removed from the wall during the time $t = t_1 + t_2$; t_1 is duration of the quasi-laminar flow over this area or the low velocity streaks, t_2 is the burst duration; Q_1 is the amount of heat removed from the wall during the period of the quasi-laminar flow t_1; Q_2 is the amount of heat removed from the wall by burst during the time t_2.

The total amount of heat removed from the wall during the time t is customarily written as:

$$Q = \alpha(T_w - T_\infty) \times s \times t \tag{2}$$

where α is the heat transfer coefficient, T_w and T_∞ are the temperatures of the wall and fluid in the free stream, respectively.

The amount of heat removed from the wall in the quasi-laminar regime is

$$Q_1 = \lambda\left(\frac{dT}{dy}\right)_{y=0} \times s \times t_1 \tag{3}$$

where λ is the thermal conductivity of the fluid and y is the coordinate normal to the wall.

To estimate the amount of heat removed by a burst of a coherent structure from the wall, we propose the following model: suppose the coherent structure leaves the wall as an axially symmetrical jet of radius ξ_o, which has a centerline at some location x_b. We reiterate that the shape which is assumed here for the burst is unimportant to the end result. The velocity distribution and temperature distribution of the jet can later be assumed to have another shape, but the results are quite independent on this assumption.

The energy which is transferred from the wall in the coherent structure is:

$$Q_2 = \left(2\pi \int_o^{\xi_o} \rho c_p v \Delta T \xi \, d\xi\right) t_2 \tag{4}$$

where $\xi = x - x_b$, x_b is the coordinate of the coherent structure, $\xi_o = (s/\pi)^{0.5}$; ρ is the fluid density, c_p is the heat capacity and v is the velocity away from the wall.

We rearrange equation (4) as follows:

$$Q_2 = \left(2\rho c_p v_m \Delta T_m \int_0^1 \frac{v}{v_m}\frac{\Delta T}{\Delta T_m} \eta \, d\eta\right) \times s \times t_2 \tag{5}$$

where v_m and T_m are the velocity and temperature at the axis of the jet, $\Delta T = T - T_\infty$. $\Delta T_m = T_m - T_\infty$.

As assumed above, that in the central part of the burst the flow is upwards along the y axis (Fig. 1). In the periphery of the burst, the direction of flow is opposite: the fluid moves towards the wall, as follows from continuity. Since the burst develops practically in the bulk of the fluid, it may be modelled as a submerged jet. Using the results of the theory of turbulent jets of Abramovich [6] we assume the following forms of velocity and temperature profiles in the burst

$$v/v_m = (1 - \eta^{3/2})^2 \quad \Delta T/\Delta T_m = (1 - \eta^{3/2})^{2Pr} \tag{6}$$

where Pr is the Prandtl number.

Substitution of (6) in (5) yields

$$Q_2 = 2\rho c_p v_m \Delta T_m s t_2 I \tag{7}$$

where

$$I = \int_0^1 (1 - \eta^{3/2})^{2(1+Pr)} \eta \, d\eta.$$

The integral in the expression (7) is a function of the Prandtl number. This integral was evaluated numerically and the results are approximated (with accuracy of 12%) by:

$$I = A/Pr^n \tag{8}$$

where $A = 0.0667$; $n = 0.155$ for $0.01 < Pr < 0.1$; $n = 0.2$ for $0.1 < Pr < 0.7$; $n = 0.57$ for $0.7 < Pr < 3$; $n = 0.8$ for $3 < Pr < 8$.

Taking into account equations (2), (3), (7) and (8) we can write the thermal balance equation (1) in the form

$$\alpha(T_w - T_\infty) = 2\rho c_p v_m \Delta T_m (A/Pr^n)\gamma + \lambda\left(\frac{dT}{dy}\right)_{y=0}(1-\gamma) \tag{9}$$

where $\gamma = t_2/t$.

Multiplying the left and right sides of equation (9) by $L/\lambda\ (T_w - T_\infty)$ and assuming that $\Delta T_m = (T_w - T_\infty)$ and v_m is proportional to the free velocity away from the wall ($v_m = \varepsilon u_\infty$, $\varepsilon < 1$) we get at $\gamma \ll 1$ the following expression

$$Nu = Nu_l + 2A\varepsilon Pr^{1-n} Re\gamma \tag{10}$$

where L is a characteristic length and where

$$Nu = \frac{\alpha L}{\lambda} \quad Nu_l = \frac{L}{(T_w - T_\infty)}\left(\frac{dT}{dy}\right)_{y=0}.$$

Nu_l is the Nusselt number for the quasi-laminar flow, i.e. for the low velocity streaks.

To estimate the value of the parameter γ, i.e. the dimensionless burst duration, we consider a number of investigations in which an average time between bursts was measured (Kline *et al.* [4], Kaftori *et al.* [5], Kim *et al.* [7], Blackwelder and Haritonidis [8], Komori *et al.* [9], etc.). These data show that the nondimensional average time between bursts

$t^+ = t(u^{*2}/\nu) = 91.5$ (u^* is the friction velocity, ν is the kinematic viscosity) approximately is a constant.

At present, data on the value of t_2 are unavailable. To estimate t_2 we use the following dimensional considerations. From the physical point of view it is clear that t_2 should be dependent on the 'outer' parameters of the flow. Therefore, we can write

$$t_2 = b\nu/u_\infty^2 \tag{11}$$

where b is an unknown empirical coefficient.

From equation (11) and the expression for the friction velocity on a plate (Schlichting [10])

$$(u^*/u_\infty)^2 = 0.0296\,Re_x^{-0.2} \tag{12}$$

with $Re_x = u_\infty x/\nu$ (x is longitudinal coordinate) we obtain

$$\gamma = \beta Re_x^{-0.2} \tag{13}$$

where

$$\beta = \frac{b}{91.5}\cdot 0.0296.$$

Substitution of (13) in the thermal balance equation (10) yields the following expression for the local Nusselt number (at $L = x$)

$$Nu_x = Nu_{xl} + BPr^{1-n}Re_x^{0.8} \tag{14}$$

where

$$Nu_x = \frac{\alpha x}{\lambda} \quad Nu_{xl} = \frac{x}{(T_w - T_\infty)}\left(\frac{dT}{dy}\right)_{y=0}$$

and $B = 2A\beta\varepsilon$ is a constant.

Taking into account the value of n in the expression (8) we obtain the following correlations for the Nusselt number at various values of Pr†

$$Nu_x = Nu_{xl} + BP_x^{0.8} \quad \text{for } 0.01 < Pr < 0.7 \tag{15}$$

$$Nu_x = Nu_{xl} + BPr^{0.43}Re_x^{0.8} \quad \text{for } 0.7 < Pr < 3 \tag{16}$$

and

$$Nu_x = Nu_{xl} + BPr^{0.2}Re^{0.8} \quad \text{for } 3 < Pr < 8 \tag{17}$$

where Pe is the Peclet number.

The first of these correlations corresponds to flows of fluid with small thermal conductivity (for example, liquid metal); the second and the third ones may be applied, for example to flows of air and water, respectively. It is seen that equations (15)–(17) agree fairly well with the known expressions describing numerous experimental data on heat removal in turbulent boundary layers at various Prandtl numbers. Since the dependence Nu_{xl} on Re_x is comparatively weak ($Nu_{xl} \sim Re_x^{0.5}$) in the quasi-laminar flow, the second term on the r.h.s. in equations (15)–(17) is the dominant one, at large values of Re_x. Hence, in fully developed turbulent flow the basic role in the heat removal from a solid wall to a fluid is played by the bursting process. For example, at the flow of water in a channel at $x = 3$ m, $u = 0.1$ m s^{-1}, $\nu = 10^{-6}$ m^2 s^{-1} the ratio of the first to the second terms in equation (17) approximately is equal to 0.3.‡

† Similarly, we can find correlations for the Nusselt number in a pipe flow (assuming $L = d$, d is the pipe diameter and using the corresponding expression for the friction velocity).

‡ To obtain this estimate we used the expression of Pohlhausen [11] for Nu_{xl} and the experimental value of the coefficient $B = 0.0296$ for the fully developed turbulent flow.

CONCLUSIONS

We propose a simple model of heat removal from the wall in a turbulent boundary layer. This model is based on the assumption that the dominant mechanism of heat transfer from the wall is the bursting of coherent structures from the wall region into the mainstream. Indeed, we assume that bursts lead to the emergence of special zones with very small thermal resistance in the near wall flow, which determine the heat transfer intensity. This model leads to the results which agree fairly well with numerous experimental data on heat removal in turbulent boundary layer at various values of the Prandtl number. The approach developed here may be used as a foundation for a theory of heat transfer in turbulent boundary layer consistent with the known experimental evidences on the internal mechanism of the flow.

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